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THE ROLE OF FINAL STATE INTERACTIONS IN ε'/ε

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The Standard Model prediction for ε'/ε is updated, taking into account the chiral loop corrections induced by final state interactions. The resulting value, $\varepsilon'/\varepsilon = (17 \pm 6) \times 10^{-4}$, is in good agreement with present measurements.

1. Introduction

The CP-violating ratio ε'/ε constitutes a fundamental test for our understanding of flavour-changing phenomena. The present experimental world average,¹ $\text{Re}(\varepsilon'/\varepsilon) = (19.3 \pm 2.4) \cdot 10^{-4}$, provides clear evidence for a non-zero value and, therefore, the existence of direct CP violation.

The theoretical prediction has been rather controversial since different groups, using different models or approximations, have obtained different results.^{2–9} In terms of the $K \rightarrow \pi\pi$ isospin amplitudes, $\mathcal{A}_I = A_I e^{i\delta_I}$ ($I = 0, 2$),

$$\frac{\varepsilon'}{\varepsilon} = e^{i\Phi} \frac{\omega}{\sqrt{2}|\varepsilon|} \left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right], \quad \Phi \approx \delta_2 - \delta_0 + \frac{\pi}{4} \approx 0, \quad (1)$$

where $\omega = \text{Re}A_2/\text{Re}A_0 \approx 1/22$. The CP-conserving amplitudes $\text{Re}A_I$, their ratio ω and ε are usually set to their experimentally determined values. A theoretical calculation is then only needed for the quantities $\text{Im}A_I$.

Since $M_W \gg M_K$, there are large short-distance logarithmic contributions which can be summed up using the Operator Product Expansion and the renormalization group.^{10,11} To predict the physical amplitudes one also needs to compute long-distance hadronic matrix elements of light four-quark operators Q_i . They are usually parameterized in terms of the so-called bag parameters B_i , which measure them in units of their vacuum insertion approximation values.

To a very good approximation, the Standard Model prediction for ε'/ε can be written (up to global factors) as⁵

$$\frac{\varepsilon'}{\varepsilon} \sim \left[B_6^{(1/2)} (1 - \Omega_{IB}) - 0.4 B_8^{(3/2)} \right], \quad \Omega_{IB} = \frac{1}{\omega} \frac{(\text{Im}A_2)_{IB}}{\text{Im}A_0}. \quad (2)$$

Thus, only two operators are numerically relevant: the QCD penguin operator Q_6 governs $\text{Im}A_0$ ($\Delta I = 1/2$), while $\text{Im}A_2$ ($\Delta I = 3/2$) is dominated by the electroweak

penguin operator Q_8 . The parameter Ω_{IB} takes into account isospin breaking corrections; the value $\Omega_{IB} = 0.25$ was usually adopted in all calculations.¹² Together with $B_i \sim 1$, this produces a numerical cancellation leading to values of $\varepsilon'/\varepsilon \sim 7 \times 10^{-4}$. This number has been slightly increased by a recent Chiral Perturbation Theory (χ PT) calculation at $O(p^4)$ which finds $\Omega_{IB} = 0.16 \pm 0.03$.¹³

2. Chiral Loop Corrections

Chiral symmetry determines the low-energy hadronic realization of the operators Q_i , through a perturbative expansion in powers of momenta and quark masses. The corresponding chiral couplings can be calculated in the large- N_C limit of QCD. The usual input values $B_8^{(3/2)} \approx B_6^{(1/2)} = 1$ correspond to the lowest-order approximation in both the $1/N_C$ and χ PT expansions.

The lowest-order calculation does not provide any strong phases δ_I . Those phases originate in the final rescattering of the two pions and, therefore, are generated by higher-order chiral loops. Analyticity and unitarity require the presence of a corresponding dispersive effect in the moduli of the isospin amplitudes. Since the S-wave strong phases are quite large, specially in the isospin-zero case, one should expect large unitarity corrections.

The one-loop analyses of $K \rightarrow 2\pi$ show in fact that pion loop diagrams provide an important enhancement of the \mathcal{A}_0 amplitude.¹⁴ This chiral loop correction destroys the accidental numerical cancellation in eq. (2), generating a sizeable enhancement of the ε'/ε prediction.² The large one-loop correction to \mathcal{A}_0 has its origin in the strong final state interaction (FSI) of the two pions in S-wave, which generates large infrared logarithms involving the light pion mass.³ Using analyticity and unitarity constraints, these logarithms can be exponentiated to all orders in the chiral expansion.^{2,3} For the CP-conserving amplitudes, the result can be written as

$$\mathcal{A}_I = (M_K^2 - M_\pi^2) a_I(M_K^2) = (M_K^2 - M_\pi^2) \Omega_I(M_K^2, s_0) a_I(s_0), \quad (3)$$

where $a_I(s)$ denote reduced off-shell amplitudes with $s \equiv (p_{\pi_1} + p_{\pi_2})^2$ and

$$\Omega_I(s, s_0) \equiv e^{i\delta_I(s)} \Re_I(s, s_0) = \exp \left\{ \frac{(s - s_0)}{\pi} \int \frac{dz}{(z - s_0)} \frac{\delta_I(z)}{(z - s - i\epsilon)} \right\} \quad (4)$$

provides an evolution of $a_I(s)$ from an arbitrary low-energy point s_0 to $s = M_K^2$. The physical amplitude $a_I(M_K^2)$ is of course independent of s_0 .

Taking the chiral prediction for $\delta_I(z)$ and expanding the exponential to first order, one just reproduces the one-loop χ PT result. Eq. (4) allows us to get a much more accurate prediction, by taking s_0 low enough that the χ PT corrections to $a_I(s_0)$ are small and exponentiating the large logarithms with the Omnès factor $\Omega_I(M_K^2, s_0)$. Moreover, using the experimental phase-shifts in the dispersive integral one achieves an all-order resummation of FSI effects. The numerical accuracy of this exponentiation has been successfully tested through an analysis of the scalar pion form factor,³ which has identical FSI than \mathcal{A}_0 .

3. Numerical Predictions

At $s_0 = 0$, the chiral corrections are rather small. To a very good approximation,⁴ we can just multiply the tree-level χ PT result for $a_I(0)$ with the experimentally determined Omnès exponentials:³

$$\Re_0 \equiv \Re_0(M_K^2, 0) = 1.55 \pm 0.10, \quad \Re_2 \equiv \Re_2(M_K^2, 0) = 0.92 \pm 0.03. \quad (5)$$

Thus, $B_6^{(1/2)} \approx \Re_0 \times B_6^{(1/2)} \Big|_{N_C \rightarrow \infty} = 1.55$, $B_8^{(3/2)} \approx \Re_2 \times B_8^{(3/2)} \Big|_{N_C \rightarrow \infty} \approx 0.92$ and $\Omega_{IB} \approx 0.16 \times \Re_2 / \Re_0 = 0.09$. This agrees with the result $\Omega_{IB} = 0.08 \pm 0.05$, obtained recently with an explicit chiral loop calculation.¹⁵

The large FSI correction to the $I = 0$ amplitude gets reinforced by the mild suppression of the $I = 2$ contributions. The net effect is a large enhancement of ε'/ε by a factor 2.4, pushing the predicted central value from^{5,6} 7×10^{-4} to³ 17×10^{-4} . A more careful analysis, taking into account all hadronic and quark-mixing inputs gives the Standard Model prediction:⁴

$$\varepsilon'/\varepsilon = (17 \pm 6) \times 10^{-4}, \quad (6)$$

which compares well with the present experimental world average.

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